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THE WELFARE CONSEQUENCES OF DIFFERENT REGIMES OF OLIGOPOLISTIC COMPETITION IN A GROWING ECONOMY WITH FIRM-SPECIFIC KNOWLEDGE

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The welfare consequences of different regimes of oligopolistic competition in a growing economy with firm-specific knowledge

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Abstract

There are two sectors: a traditional goods sector with perfect competition among firms and a high-tech differentiated goods sector with imperfect competition. Firms in the latter sector engage in R&D, which is an in-house activity. There are no externalities. Oligopolistic competition in the differentiated goods sector differs with respect to toughness. Tough competition induces relatively low profit margins, a relatively small number of firms in long-run equilibrium and a relatively fast rate of growth. The welfare consequences of different market solutions are analysed by comparing the outcomes with those of a planning approach. In the market solutions consumers maximize an intertemporal utility function. In the centralized solution the planning authority takes this function as the objective to be maximized.

JEL code: 110,620

Introduction

In the new growth theory the rate of efficiency improvement is endogenous. Technological change is fed by learning effects or is related to R&D activities. The latter type of analysis has to be more specific. R&D activities can be directed at the invention of new products or at the realization of new techniques of production. In most models it is assumed that R&D activities generate in addition to the specific knowledge aimed at also a more general form of knowlegde, which is is publicly available (e.g. Romer, 1990, Grossman and Helpman, 1991). R&D induces externalities, which improve the existing knowledge base in the laboratories, so that the productivity of factors employed in research is improved. This generates a steady path of economic growth.

In this paper we discuss the implications of R&D in a somewhat different setting. It is assumed that R&D-activities besides special knowledge for the purpose intended create additional knowledge, which is internal to the firm and is used in subsequent R&D activities. These ideas correspond with empirical work stressing that firms rely on tacit knowledge and firm-specific skills (e.g. Pavitt, 1984; Dosi, 1988). Therefore growth in our approach relies on internalities instead of on externalities.

It is by now well-known, that firms engaging in R&D need profits to pay for the fixed cost involved. Succes in R&D may create partial monopoly position which can be exploited. Assuming free entry in R&D, an equilibrium may then be established, showing that the present value of these monopoly profits equals the cost of R&D output (blueprints for new technologies). Here we assume that firms have their own niche in the market which allows them to pay for traditional fixed cost as well as for R&D outlays. In the short-run firms may make profits in excess of these fixed cost or incur losses. In the long-run free entry and exit drives these excess net profits down to zero. The number of firms in the innovating industry is, therefore, endogenous. In this respect our model is similar in spirit to that of Dasgupta and Stiglitz (1980). There are however important differences as these authors assume homogeneity of products in the high-tech sector and ignore the investment aspect of outlays on R&D.

The assumptions made allow us to study differences in the toughness of competition in relation to economic growth. If the number of firms in the initial situation is relatively small, as may be realistic in high-tech sector, firms face oligopolistic competition. If competition is tough profit margins are based on the own price elasticities of demand. In a situation with soft competition, firms take the impact of their price level on market demand for the sector as a whole into account in setting mark-ups. To allow for this possibility we have to introduce a second sector in addition to the high-tech industry on which the analysis is focussed. The second sector produces traditional goods, without engaging in R&D. For this reason it can be assumed that perfect competition prevails in the traditional goods sector.

Differences in the toughness of competition induce differences in the number of firms

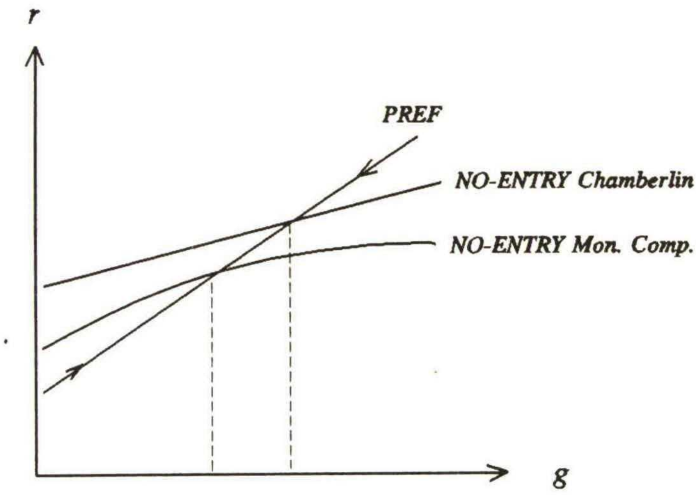

Figure 2

in a market equilibrium. There are less firms the tougher the competition is. Relatively tough competition leaves more room for allocating labour to R&D in each firm. As a result the macroeconomic growth rate will be higher.

Apart from comparing both market solutions we discuss the welfare implications of each of them. For this purpose we start with the planning solution of the model, which can be solved analytically. Competition à la Chamberlin with profit margins determined by the elasticity of substitution in the high-tech sector will be considered next. In addition, we derive results for the case in which profit margins are based on the full impact of price changes on demand for the firm's products. The differences between the solutions of the model are illustrated by a numerical example.

Feasible growth paths

There are two sectors in the economy. In the sector denoted by *Y*, firms produce traditional goods. In the sector denoted by *X*, each firm produces a differentiated product. Labour is the only production factor in the economy. By allocating labour to R&D, firms in the *X*-sector can raise labour productivity, so that the *X*-sector may be referred to as the high-tech sector. Consumers have a taste for goods from the traditional and from the high-tech sector. Moreover, consumers trade-off future consumption for present consumption.

Denoting per capita utility by U and per capita consumption by C, the intertemporal preference function can be specified as

$$U = \int_0^\infty \frac{1}{1-\rho} C_t^{1-\rho} e^{-\theta t} dt \tag{1}$$

Equation (1) is a CRRA utility function with a constant pure rate of time preference θ and a constant intertemporal elasticity of substitution $1/\rho$. The consumption index C in defined as

$$C = X^\sigma Y^{1-\sigma} \qquad 0 < \sigma < 1 \tag{2}$$

where X denotes the index of production in the high-tech sector which is given by

$$X = \left[\sum_{j=1}^{N} x_j^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \qquad \epsilon > 1 \tag{3}$$

There are N producers of differentiated products x_j. These products are imperfect substitutes with constant elasticities of substitution ϵ. Equations (1) - (3) taken together specify consumers preferences with respect to products and with respect to the time of consumption of these products.

Production of traditional goods requires one unit of labour, so that total production of

Y equals

$$Y = L_Y \tag{4}$$

where L_Y denotes the amount of labour allocated in the Y-sector. Production of a differentiated product *(i)* requires *1/h* units of labour:

$$x_i = h_i L_{xi} \tag{5}$$

where L_{xi} denotes the amount of labour applied in producing product x_i. Innovation in each firm *(i)* requires labour L_{ri} to improve labour productivity h according to the formula

$$\dot{h}_i = \xi \, h_i \, L_{ri} \tag{6}$$

where ξ is a constant parameter. The introduction of h_i on the RHS of eq. (6) captures the idea that knowledge creation is a cumulative process which feeds on existing knowledge. By assuming constant returns with respect to existing knowledge h_i, productivity growth can be sustained in the long run by allocating a constant amount of labour to R&D activities. Accumulation of knowledge is an in-house activity of each firm. It is here assumed that in-house knowledge can be kept secret. However, as shown in another paper the analysis can be enriched by allowing for knowledge spillovers between firms.[1)]

Firms in the high-tech sector incur traditional fixed cost in the form of a fixed amount of labour, f. The total amount of labour available in the economy, L, is fixed and should equal total labour demand. Assuming symmetry across firms in the differentiated products sector we can write:

$$L_Y + N\,(L_{xi} + L_{ri} + f) = L \tag{7}$$

Growth paths which satisfy equations (1) - (7) are called feasible. Which path is selected depends upon institutional arrangements. First, we consider the case where a planning authority maximizes the utility of the representative consumer. Secondly, market solutions will be analysed under different assumption with respect to the behaviour of the partial monopolist in the high-tech sector. The welfare implications of the market solutions are discussed by comparing the results with the outcomes of the centralized solution.

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1) See Smulders and Van de Klundert (1993).

The planning approach

In a planning approach the central authority maximizes eq. (1) subject to eq. (2) - (7). Because of the symmetry in the differentiated products sector it may be stated without proof that in the optimum production volumes of all differentiated products are equal to x. Instead of eq. (3) we may therefore write,

$$X = N^{\frac{\epsilon}{\epsilon - 1}} x \tag{3a}$$

Substitution of the constraints in the objective function and taking proper account of the accumulation eq. (6) gives rise to the Hamiltonian expression:

$$H = \frac{1}{1-\rho} \left\{ (N^{\epsilon/(\epsilon-1)}\, hL_x)^{\sigma(1-\rho)} \, [(L - N(L_x + L_r + f)]^{(1-\sigma)(1-\rho)} \right\} + \varphi \xi h L_r$$

where φ denotes the costate variable associated with state variable h. The instruments, which have to be chosen optimally, are N, L_x and L_r. The first order conditions with respect to these instruments can be written after some manipulations as

$$L_Y = \frac{(\epsilon - 1)(1 - \sigma)}{(\epsilon - 1) + \sigma} L \tag{8}$$

$$NL_x = \frac{\sigma}{1 - \sigma} L_Y \tag{9}$$

$$\varphi = (1 - \sigma) \left[\frac{C^{1-\rho}}{h} \right] \left[\frac{N}{\xi L_Y} \right] \tag{10}$$

Optimizing with respect to the state variable yields

$$\dot{\varphi} - \theta\varphi = -\sigma \left[\frac{C^{1-\rho}}{h} \right] - \xi L_r \varphi \tag{11}$$

The corresponding transversality condition reads:

$$\lim_{t \to \infty} e^{-\theta t} \varphi h = 0 \tag{12}$$

From eq. (8) and (9) it follows that L_Y and $L_X \equiv NL_x$ are constant. There is no growth in the traditional sector. Substitution of these result in the labour market equation (7) gives an expression for L_r. Applying the definition for the growth rate of labour productivity

$g \equiv \frac{\dot{h}}{h} = \xi L_r$ one finds:

$$g = \frac{\xi\sigma}{(\epsilon - 1) + \sigma}\,\frac{L}{N} - \xi f \tag{13}$$

Logarithmic differentiation of eq. (10) with respect to time yields

$$\frac{\dot{\varphi}}{\varphi} = (1 - \rho)\,\frac{\dot{C}}{C} - g + \frac{\dot{N}}{N} \tag{14}$$

The growth rate of aggregate consumption can be found by logarithmic differentiation of eq. (2) after subsituting $X = N^{\epsilon/(\epsilon - 1)} h L_x$ and noting that L_Y and L_x are constant:

$$\frac{\dot{C}}{C} = \frac{\sigma}{(\epsilon - 1)}\,\frac{\dot{N}}{N} + \sigma g \tag{15}$$

Equations (11), (14) and (15) can be combined to yield the following differential equation in N:

$$\dot{N} = \left[\frac{\sigma(\rho - 1)f - \theta/\xi}{\sigma(\rho - 1)/(\epsilon - 1) - 1}\right] N - \left[\frac{(\epsilon - 1)\sigma L}{(\epsilon - 1) + \sigma}\right] \tag{16}$$

Equation (16) has a positive root. The planning authority should therefore opt for a constant value of N. Otherwise N goes to infinity or will become negative. Both possibilities must be excluded on logical grounds. With N fixed the growth rate is constant as appears from eq. (13). The optimal number of firms can be obtained from eq. (16) by setting $\dot{N} = 0$.

$$N^* = \frac{(\rho\sigma + 1 - \sigma) - \epsilon}{\sigma(\rho - 1)f - \theta/\xi} \cdot \frac{\sigma L}{(\epsilon - 1) + \sigma} \tag{17}$$

Substitution of this solution in eq. (13) results in the optimal rate of growth of labour productivity in the high-tech sector:

$$g^* = \frac{(\epsilon - 1)\xi f - \theta}{(\rho\sigma + 1 - \sigma) - \epsilon} \tag{18}$$

As can easily be shown the transversality condition (12) is satisfied for $\theta > (1 - \rho)\sigma g$, which implies $f > \theta/(\rho - 1)\sigma\xi$. Economic meaningful solutions for N^* and g^* then require $(\rho - 1)\sigma > \epsilon - 1$ and $f > \theta/(\epsilon - 1)\xi$. The latter two inequalities imply the former and are therefore necessary conditions for a meaningful solution of the optimization problem. Note that the intertemporal elasticity of substitution should be smaller than unity $(\rho > 1)$. With constant Y, N and L_x the growth rate of consumption can be found by logarithmic differentiation of eq. (12): $\dot{C}/C = \sigma g^*$.

The optimal rate of growth is *negatively* related to the pure rate of time preference (θ) and *positively* related to the productivity of the R&D sector (ξ), the intertemporal elasticity of substitution $(1/\rho)$, the elasticity of substitution between differentiated products (ϵ) and the size of fixed cost (f). The impact of the first three parameters needs no further explanation. The intuition behind a rise in ϵ or f is that these changes make variety less attractive, so that the optimal number of firms declines. As a result more labour can be allocated in the remaining differentiated product sectors. Profits can be raised by increasing the amount of each good produced or by investing in productivity improvements. Starting from an optimum it pays to do both, because a rise in production (and therefore in revenue) makes R&D more attractive.

Decentralized solutions

In a market economy consumers maximize the intertemporal utility function and producers maximize the value of the firm. Prices of high-tech products are set by monopolists, while other prices and the rate of interest are determined under the conditions of perfect competition. All markets clear. On the capital market firms issue securities which are bought by households. There is no investment in physical capital goods. Firms invest in knowledge capital and use the returns to pay shareholders a dividend. Shareholders in their turn supply savings which firms need to invest.

Consumers behaviour can be formulated as a two stage budgeting problem. In the first stage, consumers decide upon the path of aggregate consumption over time by maximizing (1) subject to the dynamic budget constant

$$\dot{A}_t = r_t A_t + w_t - C_t P_{Ct} \tag{19}$$

where A, r, w and P_C denote non-human wealth, the rate of interest, the wage rate and the consumption price index respectively. From the first order conditions one can obtain by routine calculation (time subscripts are now omitted for simplification)

$$\frac{\dot{C}}{C} = \frac{(r - \dot{P}_C/P_C) - \theta}{\rho} \tag{20}$$

This is the well-known Ramsey-rule saying that households smooth consumption over time. If the real interst rate exceeds the pure rate of time preference it pays to save. In the opposite case households dissave.

In the second stage of the decision problem consumers maximize eq. (2), taking account of eq. (3), subject to the budget constraint

$$XP_X + YP_Y = CP_C \tag{21}$$

From the first order conditions one can then derive the demand equations

$$x_j = X \left[\frac{p_{xj}}{P_X} \right]^{-\epsilon} \tag{22}$$

$$Y = (1-\sigma)\frac{CP_C}{P_Y} \tag{23}$$

Substitution of these equations in the utility function yields the price indices

$$P_X = \left[\sum_{j=1}^{N} p_{xj}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{24}$$

$$P_C = \left[\frac{P_X}{\sigma}\right]^{\sigma} \left[\frac{P_Y}{1-\sigma}\right]^{1-\sigma} \tag{25}$$

Perfect competition prevails in the *Y*-sector, so that price equals cost

$$P_Y = w \tag{26}$$

Producers in the high-tech sector maximize the value of the firm by setting prices and choosing the amount of R&D to be performed. For the present value of the firm we may write

$$V = \int_0^{\infty} [x_{jt}\, p_{xjt} - (L_{xjt} + L_{rjt} + f)\, w_t\,]\, e^{-R(t)}\, dt \tag{27}$$

where $R(t) \equiv \int_0^t r(s)\, ds$ denotes the discount factor. Maximization of eq. (27) subject to the constraints (5), (6) and (22) yields after some algebraic manipulations

$$p_{xj} = \frac{e_j}{e_j - 1}\frac{w}{h_j} \tag{28}$$

$$p_{hj} = \frac{w}{\xi h_j} \tag{29}$$

$$p_{xj}\left(\frac{e_j - 1}{e_j}\right) L_{xj} + p_{hj}\, \xi L_{rj} + \dot{p}_{hj} = r\, p_{hj} \tag{30}$$

where e_j is the perceived price elasticity of demand and p_{hj} denotes the shadow price of a unit of R&D output (product-specific knowledge or blueprints). Equation (28) implies that firms set a fixed mark-up over cost. The perceived price elasticity can be derived from equations (22) and (24) as

$$e = \epsilon + (\upsilon - 1)(\epsilon - 1)\, \frac{x_j\, p_{xj}}{X P_X} \tag{31}$$

The parameter υ is introduced to capture the toughness of competition is the high-tech sector. Only polar cases $\upsilon = 0$ and $\upsilon = 1$ will be considered. If $\upsilon = 1$ competition is relatively tough so that firms may only consider the demand effects of changes in their own relative price: $e = \epsilon$. This is the assumption usually made in the Chamberlin model of monopolistic competition. If $\upsilon = 0$ firms take into account also the demand effect of a change in the aggregate price index of high-tech goods, because competition is relatively soft. The impact of a firm's price change on the aggregate index depends upon the market share, which equals $1/N$ in case of symmetry. With competition being soft the elasticity of demand, $e = \epsilon - (\epsilon - 1)/N$, is lower than in the standard Chamberlinian model and firms are able to realize higher mark-ups. We will reserve the term monopolistic competition for the latter case and speak of Chamberlinian competition if the perceived price elasticity equals ϵ.[2)]

Equation (29) implies that the marginal product of labour employed in R&D in value terms $\xi h p_h$ should be equated to the marginal cost of labour w. Equation (30) is a no arbitrage condition, saying that investing an amount of money p_{hj} in the capital market (RHS) should yield the same revenue as investing that same amount of money in knowledge production. The latter raises productivity in producing commodities and hence revenue (first term LHS), it raises the knowledge base in R&D activities (second term) and it yields a capital gain (last term).

Assuming symmetry in the high-tech sector, labour market equilibrium is given by eq. (7). In the appendix it is shown that under perfect foresight of agents the allocation of labour across sectors and firms is constant over time. The system jumps to a steady state growth equilibrium as there are no historical determined rigidities. Labour productivity in the high-tech sector grows at a constant rate, g. Under these circumstances the shadow price of blueprints, p_h, will be constant too. As appears from eq. (29), wages rise at the same rate as labour productivity in the high-tech sector. The price of good Y increases at the rate g.

Substitution of eq. (28) in eq. (29), taking account of eq. (6) and $\dot{p}_h = 0$, results in

$$L_{rj} = \left[\frac{g}{r - g}\right] L_{xj} \tag{32}$$

The ratio of labour in R&D activities to labour in direct production is higher the higher is the equilibrium growth rate and the lower is the equilibrium rate of interest. For a meaningful solution we must have $r > g$. As will be shown below, this condition will be met if the same inequality restrictions on parameters are postulated as in the planning approach.

Equations (2) - (7), (23) - (26), (28) and (32) can be applied to derive the rate of return, r, in terms of the equilibrium growth rate, g:

$$r = \xi \left[\frac{(e-1)\sigma}{e-\sigma}\right] \left[\frac{L}{N} - f\right] + \frac{e(1-\sigma)}{e-\sigma}\, g \tag{33}$$

2) See also Yang and Heijdra (1993).

Eq. (33) will be labeled the TECHnology line and is shown in Figure 1. The rate of return depends *positively* on the productivity of R&D (ξ), the availability of labour (L/N), and the taste for high-tech goods (σ). The rate of return depends *negatively* on the amount of fixed cost (f). The TECH-line has a positive slope. A rise in the growth rate implies an increase in fixed cost for R&D. Profits fall, but as a part $(1 - \sigma)$ of profits is spent on the consumption of Y-goods, this leakage declines and sales and returns in the differentiated goods sector increase.

The required rate of return by households follows from rewriting the Ramsey condition (20), noting that in the steady state $\dot{C}/C = \sigma g$ and $\dot{P}_C/P_C = (1 - \sigma)g$.

$$r = \theta + (\rho\sigma + 1 - \sigma)g \tag{34}$$

This relation is depicted by de PREFerence line in Figure 1. The relation between the required reward on savings (r) and the growth rate is positive for two reasons. First, marginal utility is declining in consumption $(\rho > 0)$, intertemporal subsitution is imperfect. Households intend to smooth consumption over time unless they are compensated by a high rate of return on savings. They ask a rate of return with a premium of $\rho\sigma g$ for postponement of consumption. Second, marginal utility is declining in X for a constant level of Y, intertemporal substitution between X and Y is imperfect. Households ask a premium of $(1 - \sigma)g$ to be compensated for the loss in utility if the X-component in aggregate consumption grows at a rate g.

Insert Figure 1

The equilibrium rate of growth for a given number of differentiated products is found at the intersection of the TECH-line and PREF-line, as shown in Figure 1. It is assumed here that the slope of the PREF-line is steeper than that of the TECH-line: $\rho\sigma + 1 - \sigma > e(1 - \sigma)/(e - \sigma)$ which condition holds for $\rho > 1$. The solution for the rate of growth is

$$g = \frac{\xi\delta(L/N - f) - \theta}{\rho\sigma + \delta - \sigma} \tag{35}$$

where $\delta \equiv (e-1)\sigma/(e-\sigma)$. The growth rate depends *negatively* on the pure rate of time preference (θ), the amount of fixed cost (f) and the number of firms (N). The growth rate depends *positively* on productivity of R&D (ξ), the intertemporal elasticity of substitution $(1/\rho)$ and the size of the economy (L).

Assuming free entry and exit of firms in the differentiated goods sector eq. (35) does not represent a long-run equilibrium, because profits (losses) will in general differ from zero. Positive profits induce firms to enter.

To simplify the dynamics we assume that *entrants* can start producing with the same technology as *incumbents*. Losses lead to exit of incumbents until a zero profit situation is realized. In the long run the number of firms in the differentiated goods sector is endogenous and can be derived from the zero profit condition $xp_x - (L_x + L_r + f)\, w = 0$. The number of firms depend on the rate of growth according to the implicit relation

$$N = \frac{\xi \sigma L}{e(g + \xi f)} \tag{36}$$

Equations (33), (34) and (36) can be solved for N, g and r. Let us consider the case of competition à la Chamberlin $(e = \epsilon)$ first. Substitution of eq. (36) in eq. (33) gives the no-entry rate of return

$$r = \epsilon\, (g + \xi f) - \xi f \tag{37}$$

Insert Figure 2

The NO-ENTRY curve is shown in Figure 2 together with the PREF line. At the point of intersection one finds the solutions for r and g

$$g = \frac{(\epsilon - 1)\, \xi f - \theta}{\rho\sigma + (1 - \sigma) - \epsilon} \tag{38}$$

$$r = \frac{[\rho\sigma + (1 - \sigma)]\, (\epsilon - 1)\xi f - \epsilon\theta}{\rho\sigma + (1 - \sigma) - \epsilon} \tag{39}$$

The solution for N is obtained by substituting eq. (38) into eq. (36):

$$N = \frac{(\rho\sigma + 1 - \sigma) - \epsilon}{(\rho - 1)\sigma f - \theta/\xi}\, \frac{\sigma L}{\epsilon} \tag{40}$$

Economic meaningful results are obtained under the same conditions as in the centralized solution: $\rho\sigma + 1 - \sigma > \epsilon$ and $f > \theta/(\epsilon-1)\xi$. As can be easily checked these inequalities imply $\rho > 1$ and $r > g$. The condition $\rho\sigma + 1 - \sigma > \epsilon$ can be interpreted as a stability condition. It implies that the NO-ENTRY curve cuts the Ramsey curve from above. Now suppose that this condition is violated and we are in a situation with a number of firms higher than in the long-run equilibrium so that the rate of growth is lower than in equilibrium. The short-run solution lies then above the NO-ENTRY line and firms realize positive profits. Free entry will lead to a larger number of firms. Hence firms in the high-tech sector receive less revenue to cover fixed cost. Profits fall and there is an incentive to cut R&D outlays on balance. This leads to a rise in profits and a further increase in the number of firms if $\epsilon > \rho\sigma + 1 - \sigma$. A relative large price elasticity induces small price reductions if output increase under impact of a reallocation of labour from R&D to production. The rate of interest does not fall much if g declines and $\rho\sigma + (1 - \sigma)$ is small. So, there is no strong incentive to raise R&D activities. The stability condition is an example of a more general condition called DSD (downward sloping demand) by Novshek and Sonnenschein (1987). The DSD-condition, which goes back to Walras, implies that in a full Arrow-Debreu equilibrium additional entry leads to new input and output prices at which the entrants make a loss. Here the DSD-condition is satisfied if the NO-ENTRY line cuts the Ramsey curve from above. The condition on f shows that these fixed cost are essential in the model.

Next, we consider the case of monopolistic competition with $e = \epsilon - (\epsilon - 1)/N$. Substitution of this price elasticity in eq. (36) yields

$$N = \frac{\sigma L \xi}{\epsilon(g + \xi f)} + \frac{\epsilon - 1}{\epsilon} \tag{41}$$

The NO-ENTRY condition can then be obtained by substitution of eq. (41) into eq. (33)

$$r = \frac{\epsilon(g + \xi f)}{1 + (\epsilon - 1)(g + \xi f)/\xi\sigma L} - \xi f \tag{42}$$

The NO-ENTRY condtition (41) is non linear as shown in Figure 2. The curve lies below the NO-ENTRY condition of Chamberlinian competition as inspection of eqs. (37) and (42) reveals. The solution for the rate of growth under monopolistic competition can be found at the point of intersection of the NO-ENTRY curve (42) and the PREF curve (34), as illustrated in Figure 2. Because of the non-linearity the explicit formula for g is not very revealing.[3] However, it is evident that the economy grows faster the tougher competition is. If competition is relatively less tough as in the case of monopolistic competition, relatively high profit margins attract more firms in long-run equilibrium. As a result aggregate fixed costs, Nf, rise and there is less labour available for production and R&D activity.

If consumers become more thrifty (decline in θ or ρ), the PREF-line moves downwards and growth increases. An increase in the productivity of R&D, an increase in the elasticity of substitution (ϵ), or a fall in fixed cost (f) shifts the NO-ENTRY curve upwards in both models of competition, so that the growth rate rises. In all cases considered the growth rate rises (falls), because the number of firms falls (rises). A change in the share

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[3] There may be two equilibrium instead of one in this case. If there are two equilibrium solutions the one with the lower growth rate is unstable, because it violates the DSD-condition of Novshek and Sonnenschein (1987).

of labour allocated in the X-sector (σL) leads to a proportionate change in the number of firms under Chamberlinian competition, and has no impact on the rate of growth. In the case of monopolistic competition the number of firms changes less than proportionate and the growth rate is affected correspondingly.

Welfare implications

The consequences for welfare of the market solutions can be discussed by comparing the outcomes with those of the planning approach. Inspection of equations (18) and (38) reveals that under Chamberlinian competition the growth rate is equal to the optimal rate of growth. However, the number of firms in the high-tech sector is too low under the market solution. The reason for this deviation from the optimum is mark-up pricing in the differentiated products sector. Because prices of differentiated goods do not reflect marginal cost, households spend too little on differentiated products and too much on traditional products (cf. Dixit and Stiglitz 1977). In the market solution relative prices do not reflect marginal costs. In the planning approach commodities are implicitely priced at their marginal cost, as can be shown easily. Denoting discounted marginal utilities of commodities x and Y by respectively U_x and U_Y central planning implies $U_x/U_Y = 1/h$ as can be checked by differentiating instantaneous utility with respect to x and Y taking into account equations (3) , (4), (5) and (7). In contrast the market economy generates:
$U_x/U_Y = p_x/P_Y = ((e-1)/e)(1/h)$

As labour allocated in the traditional sector is lower than in the centralized case, the number of firms in market equilibrium will be less than in the planning approach. The number of firms falls in the same proportion as the aggregate amount of labour in the X-sector, so that the growth rate is not affected.

Under monopolistic competition the mark-up in the differentiated goods sector is higher than under Chamberlinian competition. As a consequence, households demand less of each differentiated product in relation to the amount of Y goods consumed. It should be noted that the amount of labour in the Y-sector does not depend on the mark-up in the cases considered. With zero profits because of free entry in the differentiated goods sector we get: $L_Y = \sigma L$. As a consequence the amount of labour in the X-sector does not change either: $N(L_x + L_r + f)$ is constant. However, because consumers want to consume less of each differentiated product compared with Chamberlinian competition L_x should also decline. The balance is struck by a larger number of firms in case of monopolistic competition compared with Chamberlinian competition. Nevertheless, the number of firms is still sub-optimal as follows from a comparison with the result of the planning approach. Because R&D is now lower, the rate of growth is sub-optimal.

It is instructive to illustrate the result numerically. For this purpose the parameters of the model will be given the following plausible values:

$\theta = 0.05,\ \rho = 4,\ \sigma = 0.5,\ \epsilon = 2$
$\xi = 0.1\ f = 1$

The scale of the economy is set at $L = 100$. As can be verified, with this set of parameters the conditions for meaningful solutions given above are met. The results are presented in Table 1.

Talbe 1: Numerical solutions

model / variable	planning approach	Chamberlinian competition	monopolistic competition
$r(\%)$	-	30	24
$g(\%)$	10	10	7,6
L_x	1	2	1,64
L_r	1	1	0,76
L_Y	33 1/3	50	50
N [a]	22 2/9	12,5	14,7

[a] The integer problem is ignored for convenience.

It should be noted that the distortions of the optimum in both market solutions are due to monopolistic price setting in the high-tech sector. There are no externalities, which should be internalized. Learning by doing is an in-house activity of firms, which is taken care of in maximizing the value of the firm.

Conclusions

Toughness of competition is conducive to economic growth. If profit margins are relatively low less firms will find a place in the market, so that the amount of labour available for each firm is larger. Firms allocate a share of the additional labour to their R&D division, which results in a higher rate of growth of efficiency in the differentiated goods sector. The wage rate in the X-sector rises. Workers in the traditional goods sector benefit to the same extent as the terms of trade move in favour of the Y-sector.

Under Chamberlinian competition the profit margin depends on the elasticity of substitution between differentiated products. From a welfare point of view the rate of growth is then optimal, but the number of firms and products is too small. Welfare distortions are caused by the deviation of prices form marginal cost in the *X*-sector. As a consequence, consumers spend too much on traditional products. However, the intertemporal trade-off between present and future consumption of differentiated products is not affected by price setting in the high-tech sector. The situation is different when profit margins depend on the elasticity of substitution between differentiated products as well as on the elasticity of substitution between differentiated products and traditional products. In the latter case the intertemporal optimality conditions are also affected by imperfect competition in the *X*-sector. Therefore, under monopolistic competition the rate of economic growth and the number of firms are both sub-optimal.

In the present model, welfare distortions only come from imperfect competition in the high-tech sector. There are no externalities assumed. Introduction of knowledge spillovers between firms can easily be introduced, but complicate the analysis substantially. The welfare implications are nevertheless clear. Firms will spend too little on R&D if they can not appropriate the full return of the investment.

Appendix

In this appendix we derive the dynamic solution for the market economy with a given number of firms, N. To simplify the exposition we take the output of the representative good in the high-tech sector as the *numéraire*, so that $p_x = 1$ and $\dot{p}_x = 0$. Our aim is to derive a differential equation in L_x by proper substitutions into the Ramsey equation (20).

Given our choice of the *numéraire* eqs. (28), (29) and (30) imply

$$\xi L_x + g = r \tag{A.1}$$

From eqs. (2) - (4) and the first order conditions (22) - (25) it is easy to derive under the condition of symmetry in the high-tech sector

$$L_Y = \left[\frac{1-\sigma}{\sigma} \frac{e}{e-1} \right] N L_x \tag{A.2}$$

Substitution of eqs. (6) and (A.2) in the labour market relationship (7) results in

$$\frac{e-\sigma}{\sigma(e-1)} L_x + \frac{g}{\xi} = \frac{L}{N} - f \tag{A.3}$$

Combining (A.1) and (A.3) one gets

$$r = -\frac{\xi e(1-\sigma)}{\sigma(e-1)} L_x + \xi \left[\frac{L}{N} - f \right] \tag{A.4}$$

Logarithmic differentiation with respect to time of eqs. (26) and (28), given our choice of the *numéraire*, results in $\dot{w}/w = \dot{P}_Y/P_Y = g$. Taking account of this result logarithmical differentiation with respect to time of eq. (25) gives

$$\frac{\dot{P}_C}{P_C} = (1 - \sigma)g \tag{A.5}$$

Substitution of eqs. (3), (4) and (5) into (2) and differentiating logarithmically with respect to time yields

$$\frac{\dot{C}}{C} = \sigma g + \frac{\dot{L}_x}{L_x} \tag{A.6}$$

Substitution of (A.4), (A.5) and (A.6) in the Ramsey rule results in the differential equation

$$\dot{L}_x = \frac{\xi}{\rho} \frac{(e - \sigma)}{\sigma(e - 1)} \left[\sigma\rho + (1 - \sigma) - \frac{e(1 - \sigma)}{e - \sigma}\right] L_x^2 - \left[\frac{\xi\sigma(\rho - 1)}{\rho}\left\{\frac{L}{N} - f\right\} - \frac{\theta}{\rho}\right] L_x \tag{A.7}$$

Possible solutions are illustrated in Figures I and II. In both cases there

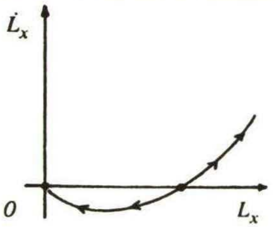

Figure I: $\rho\sigma + (1 - \sigma) > \dfrac{e(1 - \sigma)}{e - \sigma}$

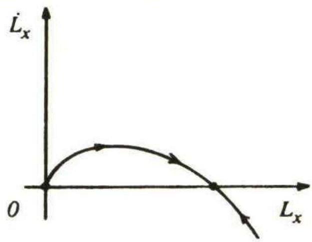

Figure II: $\rho\sigma + (1 - \sigma) < \dfrac{e(1 - \sigma)}{e - \sigma}$

is a degenerate solution at $L_x = 0$. For $\rho\sigma + (1 - \sigma) > e(1 - \sigma)/e - \sigma$ the second solution for L_x is unstable. Therefore, under perfect foresight economic agents will jump to this solution. Only the stationary solution can be an equilibrium because any other solution would lead to a negative L_x or to L_x larger than L. In the case of Figure II with $\rho\sigma + (1 - \sigma) < e(1 - \sigma)/e - \sigma$ the second solution is stable. There is now a continumum of initial values for L_x consistent with a convergent solution. As argued by Buiter (1984), the stationarity property can be imposed in this case, because the system with one forward looking variable and one root should not depend on "irrelevant" past values. Nevertheless, the chosen solution of the fastest convergence is arbitrary, as is every other possibility. From an empirical point of view the picture shown Figure II is less likely to emerge as it requires $\rho < 1$.

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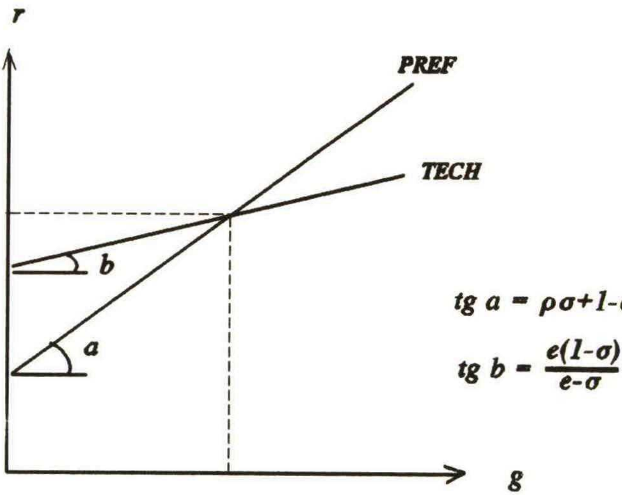

Figure 1

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